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Maddy's *Realism in Mathematics* is a noble endeavor to solve some of the ancient dilemmas encountered in the philosophy of mathematics. Although the work has since been abandoned by Maddy herself, I will endeavor to show that it is a worthy attempt to solve the problems of mathematical metaphysics and epistemology despite some fundamental flaws both in its intention and execution. The work is worthy because its attempted solution to these problems seeks to do justice to the manner in which mathematicians themselves perceive their art. Maddy's realism may possess many philosophical problems in and of itself, but I believe its most damning aspect lies in its compromising attitude towards the very Platonism to which it is trying to pay homage. Indeed, some of the philosophical failings of *Realism in Mathematics* to which Maddy herself is blind (e.g., combining empiricism and realism) are addressed in the writings of Plato himself. Maddy's project should not have been abandoned for Naturalism, but rather for a stronger Platonistic realism.

Maddy explains from the very outset of her project that her motivation for choosing a realist metaphysics is that most mathematicians are platonistic realists. They believe that they are discovering new properties in independently existent "real" entitites. Some of the greatest mathematicians in recent times were realists, such as Frege and Gödel, and as the bearers of the greatest mathematical knowledge, we should very likely trust their intuitions. I will return to the Platonism of mathematicians later in the paper, but first I wish to explore Maddy's compromise of Platonism in parallel with the ideas put forth by Plato himself in his dialogue *Theaetetus*. I will conclude with some of the failings Maddy's Platonism incurred, I believe, by the compromise.

1 - Plato

Ostensibly, the Platonic dialogue *Theaetetus* concerns the nature of knowledge. However, one should be open to interpretations of the dialogue which concern mathematical knowledge in particular. The two principle characters, Theodorus and Theaetetus, are both mathematicians, and their dialogue is introduced as being read to Euclid. There will probably be at least a few instances in which I am perhaps attributing too much to the *Theaetetus*, but I believe a strength of the Platonic format is that it is loose enough to inspire new philosophical contemplation in its readers.

The dialogue begins with the assertion that knowledge is perception, and this is later amended to truly perceiving, for mistakes in perception or in memory cannot be admitted as knowledge. This is directly analogous to the Benaceraffian difficulty which prompted both Maddy's work and so much else in the philosophy of mathematics. That is, if we admit primarily empirical sources for our knowledge, then how are we to countenance mathematical objects? As abstract, acausal entities, we cannot have the kind of sensual interaction necessary to gain justified, true beliefs about them.

Plato's finesse to this kind of objection is to advocate a kind of rationalism. (I am aware of the anachronism here, but again, it is my interpretation of the dialogue.)

> You are thinking of being and not being, likeness and unlikeness, sameness and difference...and other numbers which are applied to objects of sense; and you mean to ask, through what bodily organ the soul perceives odd and even numbers and other arithmetical conceptions...my only notion is, that these, unlike objects of sense, have no separate organ, but that the mind, by a power of her own, contemplates the universals in all things.(Plato, p.18)

This seems to be a much weaker rationalism than the remembrance argument posed in the *Meno*. Certain kinds of knowledge, such as the knowledge of universals or mathematics, seems to be a result of the ruminations of our own minds. They are intrinsic aspects of our cognitive processes. This assertion appears to skirt possible objections from Quinean arguments against aprioricity. While such objections may have their place, it is my belief that they are taken too strongly. Within both the philosophy of language and the philosophy of mathematics, there is ample reason to believe that we have, built into the very operations of our mind, the ability to create languages, mathematical assertions and universalize particulars. This would suggest that, at least as far as human knowledge is capable of delving, mathematical abstracts are intuitively accessible.

Doubts about the Quinean anti-aprioricity arguments are something I would like my reader to keep in mind, but I do not want to stray too far from the topic. My previous assertions should probably be qualified here, for Plato's locutors take a distinctly empirical position towards the end of the dialogue. Socrates asks Theaetetus to imagine the mind as an empty aviary, and the various forms of knowledge are birds which must be possessed and caught in order to be utilized. The empty aviary at birth is the empiricist's blank slate in the mind of a child.

Plato develops another empirical notion in his description of knowledge, that of fundamental elements. Plato compares these fundamental elements to the letters in a syllable, many of which make up a word. Analogously, a complex concept is made up of many smaller concepts, and these of fundamental elements. These elements must be perceived and cannot be known, as he states:

> the elements or letters are only objects of perception, and cannot be defined or known; but the syllables or combinations of them are known and expressed, and are apprehended by true opinion. (opposing knowledge - rational explanation of all the parts) (Plato, p.26)

Why then, it may rightly be wondered, do I maintain that a rational element is being implied in this very obvious empiricism? *Theaetetus* is one of Plato's dialogues that does not arrive at a firm solution. It ends by dismissing the empirical notions of knowledge because they do not allow for a conceivable method of relating generalities to particulars. That is, to have knowledge of a particular thing is to have both a true opinion of it and to apprehend those characteristics which distinguish it from all other things. However, as Socrates states,

> We are supposed to acquire a right opinion of the differences which distinguish one thing from another when we have already a right opinion of them, and so we go round and round...for to add those things which we already have, in order that we may learn what we already think, is like a soul utterly benighted. (Plato, 30)

Thus, the requirements are circular, to obtain knowledge of a thing is to be able to distinguish it by its differences, but to distinguish it by its differences is to have knowledge of a thing.

It is my belief that the failing here is supposed to make us recollect some of Plato's other writings and through a combination of ideas arrive at a solution. As previously stated, Plato's *Meno* advocates a kind of rationalism by asserting that all knowledge is remembered from a previous existence. If we weaken that conception of knowledge and combine it with the now foundering empiricism of the *Theaetetus*, we have a conception of knowledge in which most things are learned but certain abstract universals must be intrinsic to our mind, lest a vicious circle develop.

Thus, the application to mathematics now becomes more clear. Mathematics, as a study of abstract entities, is similar to Plato's conception of universals as forms. In order for philosophical or mathematical pursuits to have any kind of merit, there must be a way for the human mind to apprehend those concepts. We are quite able both to recognize instances of universals and understand mathematical applications. Recall also that the slave in Plato's *Meno* was said to "remember" the mathematical theory which Socrates is teaching from the depths of his mind/soul. The soul, as the measure of man, must be able of its own accord to obtain knowledge of abstracts such as universals or mathematical knowledge. To modernize this exercise, Maddy's empiricism should have us perceive sets because it is essential to the very workings of our mind that we do so, not because they exist in their instantiations, and a flaw of her argument is that she affirms the opposite. (Although, it could be regarded as implicit in her neuro-physiological ruminations, except that she is such a confirmed empiricist.)

2 - Maddy

My reformulation of Maddy can now be more explicitly expressed. Maddy believed that realism was justified from indispensability arguments concerning the necessity of mathematics to science. It seems to me that realism should be affirmed from the same sorts of considerations that prompts most mathematicians to believe that their field concerns independent, real objects. It is perfectly acceptable to use naturalistic justifications for our ability to perceive sets, Gödel himself appears to have believed this, but science indispensability concerns should not rule out the possibility of mathematical realism.

A brief elaboration of certain key elements in Maddy's philosophy will clarify the argument. Maddy developed a two-tiered epistemology for mathematical objects in her book, *Realism in Mathematics*. The first tier of her epistemology presents a possible method by which we might perceive mathematical objects. Maddy posits that when we are presented with physical objects, our mind develops certain faculties by which groupings of physical objects, or sets in an impure sense, might be recognized.

The cognitive theories she develops to support this claim are taken from the neurophysiologist Hebb. Briefly, Maddy and Hebb believe that when the senses are presented with various physical objects, the neurons of the mind develop into cell-assemblies which become stimulated whenever the senses are again stimulated by that object. This theory allows Maddy to satisfy the Benaceraffian difficulties of sufficient causal interaction between subject and object to allow knowledge. If we have knowledge of any physical object, then we have knowledge of sets. However, Maddy takes her epistemology to be purely empirical, so there are generally philosophical problems that arise when a realist metaphysics and an empiricist epistemology are combined.

Levine's essay, *Conjoining Mathematical Empiricism with Mathematical Realism: Maddy's Account of Set Perception Revisisted*, fleshes out some of these difficulties. Essentially, Levine states that Maddy's empiricism is too stringent, and does not allow for set perception in instances in which mathematical empiricists would like to say that such perception has occurred. For example, a Gettier case is posited in which a subject perceives a set of three chairs before him, and there are actually three chairs before him, but the objects of his perception is an image of three chairs that are behind him. In the Maddian system we are not obtaining knowledge of the set of three chairs, but rather of three chair percepts, and this only if a rather loose interpretation of the Maddian system is employed. This results, through an application of Kitcher's definition of *a priori*, in empricial *a priori* knowledge, a blatant contradiction in terms.

By adding a rationalist element into Maddy's theory, I believe that this problem can be overcome and the first tier salvaged. This also brings her realism more in accord with the original ruminations of Plato as I mentioned earlier and brings her philosophy a bit more in line with some of Gödel's thinking. That is, the developing human mind will, barring mishap, always develop certain concepts concerning groupings of objects. These truths, then, are in a sense built into the very functioning of our cognitive faculties. This would imply a degree of rationalism that Maddy does not seem to want to admit, although as previously stated it is implicit in her writings on the subject. Consider,

> Do I test to see whether or not the two sets of fingers on my right and left hands can be combined to form a larger set of fingers? No, once I am able to understand the questions the answer is obvious...In childhood, such manipulations ...helped engender my ability to see sets in the first place..but once I have this ability, my conviction...doesn't depend on my testing a variety of the sets I now see.(Maddy, 67-68)

Thus, the formation of set concepts would arise no matter what our sensory history, and they qualify as *a priori* knowledge by Kitcher's definition. By accepting that the fundamental concept is indeed an *a priori* truth about us, we introduce enough rationalism to maintain an intelligible account of set perception.

3 - Gödel

We do not, I believe, wish to make this rationalism too strong, lest we put too great a distance between our perceptual faculties and the mathematical objects whose reality we desire to maintain. Mathematical objects must be sensually apprehensible. However, the fact that these truths appear to be built into the workings of our minds helps to give credence to Gödel's beliefs about mathematical intuition. That is, it is almost as if the mind contains a sixth sense which allows us to perceive mathematical objects. The reason for his firm dedication to the reality of mathematical objects lays in his earlier works in mathematics. Gödel's proof of undecidability in formally structured mathematics led him to believe that the only true measure of mathematical knowledge must come from perceptions, albeit at times unclear, of independent mathematical entities. As Wang writes,

> In fact, his (Gödel's) experience was very gratifying when he was able to introduce the constructible sets by combining the highly nonconstructive (and indeterminate for our knowledge) concept of arbitrary ordinals with the avowedly anti-objectivistic ramified hierarchy. G himself is much aware of the flexibility of the use of his sincerely believed...objectivism (Wang, 204) [See also Appendix A]

Gödel also points to the fundamental rules of logic as apparent only to the power of intuition. [See Appendix B for Wang's elaboration of this]

Despite the merits of these arguments, I realize that they do not stringently prove the existence of mathematical objects, especially since I advocated a form of rationalism that implies the aprioricity of certain fundamental mathematical and logical truths. How are we to be sure that the Gödelian intuitionism and Platonic rationalism that I have introduced do not merely point to a fact about our cognitive processes alone and not to an ability of those processes to characterize and explore existent mathematical objects?

4 - Indispensability

One of Maddy's chief reasons for abandoning realism was a concern over indispensability's (the second tier of her epistemology) ability to justify mathematical objects as actually existent. In her work, *Realism in Mathematics*, Maddy supported the claim that the mathematical objects that we perceive are real through Quinean indispensability arguments. That is, she claimed that a realistic, objective, ontology was indispensable to our best understanding of science and maethematics. However, in her more recent work, *Naturalism in Mathematics*, she repudiates this line of thought,

> among the various justifications proposed for 0# and the rest, the most compelling seem to rest on maximizing principles of a sort quite unlike anything that turns up in the practice of natural science...the scientist posits only those entities without which she cannot account for our observations, while the set theorist posits as many entities as she can, short of inconsistency (Maddy, 131)

Maddy now dismisses indispensability's capability to justify a realist ontology. Such a claim seems to be absurd in two respects. First and foremost, it blatantly contradicts the manner in which mathematics is conducted and viewed by its practitioners. Secondly, the "irreconcilable" discrepancy between scientific and mathematic indispensability does not seem to me to exist, at least in the condemning manner in which Maddy views it.

My first objection is a common theme in this paper, and I have already discussed it at length. Mathematics without a realist ontology loses the objective claim so important to the pursuit. Furthermore, and Maddy herself seemed to have argued this point, a realist ontology is indispensable to the science of mathematics itself. Science can be "done" without math, but it is easier to complete *with* mathematics. Mathematics flows more freely, makes more sense in the minds of its practitioners, when it is *realistically* considered. That is, it is indispensable for the necessary intuitive conceptions for mathematics to be thoroughly explored that it be considered real. As Maddy states when describing the controversy over the axiom of choice,

> From our set theoretic realist's perspective on mathematical evidence, Zermelp is recognizing both intrinsic supports - in terms of 'intuitive evidence' - and extrinsic supports - in terms of the role of the axiom in overall scientific theorizing. (Maddy, 118)

Gödel as well viewed the elusive nature of set theory as a testament to the independent, objective nature of mathematics. Just as certain complex environmental systems defy clear-cut mathematic systematization (e.g. weather patterns, certain chemical reactions, certain aspects of quantum physics), mathematics itself defies strict formalization. In its most essential form, mathematics attains the kind of complexity available only to objects beyond our control with their own independent reality.

The second objection to Maddy's criticism is that it is not sufficient grounds for dismissing a realist ontology. Though I am in favor of treating mathematics as a science and opening it to the same sorts of qualifications that sciences receive, I do not believe that it can be appreciated in exactly the same manner as a strictly physical science. Although mathematics seeks ontologically rich first principles, whereas science is conservative and careful in its explorations, the nature of mathematical objects can only become observable from a sufficiently rich ontology. Scientific theories which invalidate further study, i.e. over-incorporate until they become irrefutable and sterile, are vacuous. Similarly, mathematical theories which result in too barren an ontology invalidate the ability for further mathematical questioning and study. This is merely a methodological concern and should not influence the metaphysical standing of mathematical objects. Furthermore, the overlap of science and mathematics in certain highly theoretical fields (e.g. string theory), proves to me that mathematical objects can be considered existent, and I see no principled basis why this existence should apply to some but not all of them.

5 - Conclusion

Mathematical entities are most efficient and intuitive when they are considered to be real. Historically speaking, such a claim requires tempering Maddy's empiricism with a bit of rationalism. This is not to abandon an empirical notion of mathematical knowledge, but merely to formulate a "compromise empiricism" that grants the mind the *a priori* ability to recognize sets. Set-hood in the abstract thus resides within the mind independent of particular experience, but particular experiences can aid in the development of a more robust understanding of set-hood. That is, a child will not develop the latent concepts unless sensual contact with some sets is established, and later in life visual aids may help to further understanding of more complex concepts. Quinean indispensability arguments also assist in the proof of mathematical reality, although their problematic consequences need not invalidate mathematical realism once the nature of mathematical objects is more properly understood. These concerns aside, however, it seems that simple reflection on mathematical practice and cognition is sufficient to entail realism. The mathematician is like one squinting at an object in the distance, and who in time will see it more clearly, or misrepresent it, or see it differently than another. One may easily picture Gödel in this way while he was beginning to prove the consistency of the axiom of choice, when he wrote the words, "A (Every set is constructible) seems to be absolute in some sense, although it is not possible in the present state of affairs to give a precise meaning to this phrase."(Gödel, 26)

Appendix A

Part of a letter by Gödel, reprinted in Wang's work, *Reflections on Gödel*, that illustrates the necessity of non-constructivist methods in proving the consistency of the continuum. It is relevant to our present discussion by the illustrated worth of intuitive necessity. Reproduced here due to spatial concerns within the body of the essay proper.

It must be understood *cum grano salis*. Of course, the formalistic point of view did not make *impossible* consistency proofs by means of transfinite models. It only made them much harder to discover, because they are somehow not congenial to this attitude of mind. However, as far as, in particular, the continuum hypothesis is concerned, there was a special obstacle which *really* made it *practically impossible* for constructivists to discover my consistency proof. It is the fact that the ramified hierarchy, which had been invented *expressly for constructivistic purposes*, has to be used in an *entirely nonconstructive way*. A similar remark applies to the concept of mathematical truth, where formalists considered formal demonstrability to be an *analysis* of the concept of mathematical truth and, therefore, were of course not in a position to *distinguish* the two. (Wang, 204-205)

Appendix B

Lewis Carrol's "What the tortoise said to Achilles," reproduced by Wang to help illustrate the difference between intuition and proof.

(A) Things that are equal to the same are equal to each other.
(B) The Two sides of this triangle are things that are equal to the same
(C) The two sides of this triangle are equal to each other.
(D) If A and B are true, Z must be true.
(E) If A and B and C are true, Z must be true.
(F) If A and B and C and D are true, Z must be true.

And so on.

The Solution to this puzzle is the observation that we stop at (C), because our logical intuition tells us that (C) is true...which we do not prove but see to be true by intuition. (Wang, 204)

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